

# Quantum Machine Learning: Algorithms, Applications, and Limitations

# Abstract

As the limits of classical machine learning (ML) become increasingly evident—especially in terms of computational scalability and data processing—quantum computing emerges as a promising frontier capable of addressing these challenges. **Quantum Machine Learning (QML)** represents the intersection of quantum computing and traditional ML, aiming to leverage quantum phenomena such as superposition, entanglement, and quantum parallelism to enhance learning capabilities, accelerate training, and solve complex problems that are intractable for classical algorithms.

This paper provides a comprehensive overview of the evolving field of QML, beginning with the theoretical foundations of quantum computing and its synergy with classical machine learning principles. We explore key QML algorithms, including quantum support vector machines (QSVM), variational quantum classifiers (VQC), quantum neural networks (QNN), and hybrid quantum-classical approaches that exemplify the practical integration of both paradigms.

We further examine real-world applications of QML across domains such as drug discovery, finance, climate modeling, and optimization, where quantum enhancements have begun to show early but promising results. Despite its theoretical potential, QML faces substantial limitations, including quantum hardware instability, limited qubit scalability, noise sensitivity, and unresolved challenges in quantum data encoding and interpretability.

In synthesizing recent advancements and persistent obstacles, this paper highlights the importance of continued interdisciplinary research, algorithmic refinement, and hardware evolution. We also outline future directions that could lead QML from experimental promise to practical utility, including developments in quantum error correction, benchmarking standards, and application-specific quantum architectures.

# Keywords

Quantum Computing, Machine Learning, Quantum Algorithms, Variational Circuits, QML Applications, Limitations

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# 1. Introduction

Over the past two decades, **machine learning (ML)** has become a foundational tool in data-driven decision-making across industries including healthcare, finance, logistics, and scientific research. Traditional ML algorithms have demonstrated significant success in tasks such as pattern recognition, predictive modeling, and natural language processing. However, as data complexity and volume continue to grow exponentially, classical computational methods face serious **limitations** in terms of scalability, speed, and efficiency. Problems involving high-dimensional datasets, combinatorial optimization, and non-convex function landscapes often stretch the limits of classical computing power, leading to increasing interest in novel computational paradigms.

**Quantum computing**, once considered a theoretical novelty, has rapidly advanced to the point where it can provide practical computational advantages for specific problems. Leveraging principles such as **superposition**, **entanglement**, and **quantum interference**, quantum computers can, in theory, perform certain calculations exponentially faster than their classical counterparts. This potential is particularly appealing for ML, where the complexity of data and models often creates bottlenecks in training and inference.

Quantum Machine Learning (QML) sits at the intersection of these two fields, combining quantum information processing with machine learning models to explore new frontiers in algorithmic efficiency and problem-solving. By encoding data into quantum states and exploiting the computational advantages of quantum operations, QML aims to speed up tasks such as data classification, clustering, regression, and reinforcement learning. Early demonstrations using hybrid quantum-classical architectures, such as variational quantum circuits, have shown that even today's noisy intermediate-scale quantum (NISQ) devices may offer advantages in specific scenarios.

Despite the promise, QML remains a **nascent and rapidly evolving field**. There are several unresolved challenges: quantum hardware is still limited by noise and qubit count; algorithm development lacks standardization; and many real-world use cases have yet to demonstrate tangible quantum advantage over classical ML. Moreover, quantum data encoding, interpretability of quantum models, and the integration of quantum algorithms into existing workflows present significant technical and conceptual hurdles.

This paper aims to provide a **comprehensive exploration** of the current state of QML, structured as follows:

- We begin by reviewing the **theoretical background** of both quantum computing and classical ML to build foundational context.
- Next, we present a detailed overview of major **quantum machine learning algorithms**, including both supervised and unsupervised approaches.
- We then examine key **application areas**, ranging from scientific research and drug discovery to finance and supply chain optimization.
- This is followed by a discussion of the **challenges and limitations** facing QML, particularly those stemming from quantum hardware constraints and algorithmic maturity.



• Finally, we explore emerging **future directions**, outlining key research opportunities and technological milestones that must be reached to realize the full potential of QML.

By analyzing the state-of-the-art in both theory and application, this paper aims to bridge the gap between the **hype and reality** of QML, offering a balanced perspective for researchers, technologists, and stakeholders in the quantum computing ecosystem.

# 2. Background and Theoretical Foundations

To understand the scope and promise of Quantum Machine Learning (QML), it is essential to establish a foundational understanding of both **quantum computing** and **classical machine learning (ML)**. This section presents the core concepts of these two fields and explains how their intersection leads to the emergence of QML as a novel computational paradigm.

# 2.1 Fundamentals of Quantum Computing

Quantum computing is a field that leverages the principles of quantum mechanics to process information in fundamentally different ways than classical computers. Unlike classical bits, which represent either a 0 or a 1, **quantum bits (qubits)** can exist in a **superposition** of states, allowing them to represent both 0 and 1 simultaneously.

Key quantum properties include:

- Superposition: A qubit can exist in a combination of basis states  $|0\rangle|0\rangle$  and  $|1\rangle|1\rangle$  angle  $|1\rangle$ , represented as  $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle|\rangle$  where  $\alpha = \langle alpha|0\rangle$  and  $|1\rangle|1\rangle$ , where  $\alpha = \langle alpha|0\rangle$  and  $|1\rangle|1\rangle$ , where  $\alpha = \langle alpha|0\rangle$  and  $|1\rangle|1\rangle$  and  $|1\rangle|1\rangle$ , where  $\alpha = \langle alpha|0\rangle$  and  $|1\rangle|1\rangle$  and  $|1\rangle|1\rangle$ .
- **Entanglement**: Multiple qubits can be linked in such a way that the state of one qubit directly affects the state of another, regardless of distance. This non-classical correlation enables exponential growth in the computational state space.
- **Quantum Interference**: Quantum algorithms utilize constructive and destructive interference of probability amplitudes to enhance correct outcomes and suppress incorrect ones.
- **Measurement**: Observing a quantum state collapses it to one of its basis states, governed by the squared amplitude probabilities.

These principles enable quantum computers to explore complex solution spaces more efficiently than classical counterparts in specific tasks such as factoring (Shor's algorithm) or unstructured search (Grover's algorithm).

# 2.2 Basics of Classical Machine Learning

Machine learning involves training models to identify patterns and make predictions or decisions based on data. Key types of ML include:



- **Supervised Learning**: Learning a mapping from inputs to outputs using labeled data (e.g., classification, regression).
- Unsupervised Learning: Finding hidden patterns in unlabeled data (e.g., clustering, dimensionality reduction).
- **Reinforcement Learning**: Learning optimal actions through trial and error in dynamic environments.

Classical ML models often involve iterative training over large datasets and high-dimensional parameter spaces. This process becomes computationally expensive, especially with complex models like deep neural networks or large-scale support vector machines.

# 2.3 Quantum-Classical Synergy in QML

QML aims to combine the **expressive power of ML** with the **computational efficiency of quantum algorithms**. The primary motivation is to develop models that can either outperform classical ML or solve currently intractable problems. There are three main approaches:

- Quantum-enhanced ML: Uses quantum algorithms to accelerate classical ML tasks (e.g., quantum linear algebra for faster kernel methods).
- Quantum-native ML: Develops learning algorithms that run entirely on quantum systems (e.g., quantum neural networks).
- **Hybrid quantum-classical systems**: Utilize both quantum and classical resources (e.g., variational quantum circuits with classical optimizers).

# 2.4 Quantum Data and Encoding Strategies

One of the most critical aspects of QML is how classical data is translated into quantum states. Several encoding methods are used:

- **Amplitude encoding**: Encodes data into the amplitudes of quantum states (compact but hard to implement).
- Angle encoding: Maps features to qubit rotation angles (widely used in variational circuits).
- Basis encoding: Directly encodes binary strings into quantum basis states.

Efficient data encoding remains a bottleneck, as the process must preserve structure and scalability without excessive quantum overhead.

# 2.5 Theoretical Complexity and Quantum Advantage

Quantum algorithms often claim theoretical speedups over their classical counterparts. For example, the quantum Fourier transform enables exponential speedups in problems like factoring. In QML, **quantum kernel estimation**, **quantum principal component analysis (qPCA)**, and **quantum clustering** propose polynomial or exponential gains—though these advantages are often contingent on idealized conditions.



Understanding whether and when quantum advantage is **practical** remains an ongoing research question, especially in the context of noisy intermediate-scale quantum (NISQ) devices.

# 3. Quantum Machine Learning Algorithms

Quantum Machine Learning (QML) leverages quantum computational principles to enhance the efficiency and capability of traditional machine learning algorithms. By exploiting quantum properties such as superposition, entanglement, and quantum parallelism, QML algorithms aim to perform complex tasks more efficiently than their classical counterparts. This section outlines several foundational and emerging QML algorithms, categorized by learning type and architecture.

# 3.1 Quantum Supervised Learning Algorithms

Supervised learning involves training a model on labeled data to predict outcomes for new, unseen inputs. QML adapts several classical supervised models using quantum computation.

# 3.1.1 Quantum Support Vector Machine (QSVM)

QSVMs extend classical SVMs by utilizing **quantum kernels** that calculate inner products in highdimensional Hilbert spaces more efficiently. A quantum computer can potentially estimate these kernels exponentially faster than a classical system, particularly for datasets with complex feature spaces.

- *Key advantage:* Kernel estimation speed-up.
- *Example:* Havlíček et al. (2019) demonstrated a quantum-enhanced kernel method using a feature map implemented via quantum circuits.

# 3.1.2 Variational Quantum Classifier (VQC)

VQCs are hybrid algorithms that use parameterized quantum circuits (also known as **ansätze**) combined with classical optimizers. The quantum circuit processes input data, and the parameters are tuned based on a loss function (e.g., cross-entropy).

- *Key advantage:* Compatible with noisy intermediate-scale quantum (NISQ) hardware.
- Use case: Binary or multi-class classification tasks on small- to medium-sized datasets.

# 3.2 Quantum Unsupervised Learning Algorithms

Unsupervised learning focuses on discovering patterns or structures in unlabeled data.



# 3.2.1 Quantum Clustering

Quantum clustering techniques explore the use of quantum distance measures or potential-based models. One approach involves quantum-enhanced versions of k-means clustering, where the distance between points is calculated using quantum states and interference patterns.

# 3.2.2 Quantum Principal Component Analysis (qPCA)

qPCA aims to find the principal components of a density matrix by estimating the eigenvalues and eigenvectors using quantum phase estimation. It offers an exponential speedup in ideal conditions when the dataset can be efficiently encoded into a quantum state.

• *Key limitation:* Requires quantum access to the full dataset as a density matrix.

# 3.3 Quantum Neural Networks (QNNs)

Quantum Neural Networks are quantum analogs of classical neural networks. Instead of layers of neurons, QNNs consist of layers of quantum gates, where qubit states evolve through parameterized circuits.

- Quantum Circuit Learning: A general-purpose QNN framework using variational circuits to approximate functions.
- Quantum Boltzmann Machines: Based on energy-based models, where quantum states represent probabilistic distributions over solutions.

QNNs remain in early stages of development, with ongoing research on architectures, training stability, and gradient estimation (e.g., through the parameter-shift rule).

# 3.4 Quantum Reinforcement Learning (QRL)

Quantum reinforcement learning applies quantum strategies to decision-making problems. QRL agents interact with quantum environments or use quantum-enhanced policy updates to learn optimal behaviors.

• *Examples include:* Quantum agents that use Grover-like search for action optimization or quantum states to represent value functions.

QRL is largely theoretical at present but is a promising area for long-term research, especially in quantum robotics and adaptive systems.



# 3.5 Hybrid Quantum-Classical Algorithms

Given the limitations of current quantum hardware, most practical QML implementations use **hybrid frameworks**, combining quantum circuits for computational bottlenecks with classical components for tasks like optimization and data preprocessing.

- *Example frameworks:* IBM's Qiskit Machine Learning, Google's TensorFlow Quantum, and PennyLane.
- *Advantages:* Feasible on NISQ devices, modular design, and compatibility with classical ML workflows.

# 4. Applications of Quantum Machine Learning

Quantum Machine Learning (QML) is beginning to demonstrate its potential across a wide range of application domains, particularly in areas that require high-dimensional data processing, optimization, or simulation. While many of these applications are still in their early stages, research and prototype systems suggest that QML could offer computational advantages in solving complex, resource-intensive problems. This section outlines key real-world and emerging applications of QML.

# 4.1 Drug Discovery and Molecular Simulation

One of the most promising applications of QML lies in **drug discovery and material science**, where quantum computers can simulate molecular interactions more accurately than classical models.

- **Molecular modeling:** Quantum neural networks can model the behavior of molecules and predict how drugs bind to target proteins, an essential step in drug design.
- **Protein folding:** QML has been used to classify protein structures and optimize folding pathways.
- Quantum chemistry integration: Hybrid QML models can integrate quantum chemistry simulations with machine learning for faster and more precise prediction of molecular properties.

*Example:* Companies like **D-Wave**, **IBM**, and **ProteinQure** are developing quantum-enabled platforms for pharmaceutical research.

# 4.2 Financial Modeling and Risk Analysis

The financial sector is highly data-intensive and requires real-time analytics, optimization, and forecasting—all areas where QML shows potential.

- **Portfolio optimization:** Quantum algorithms can evaluate large combinatorial optimization problems such as the optimal asset allocation under risk constraints.
- **Fraud detection:** Variational classifiers can be trained to recognize patterns of fraudulent transactions with higher efficiency.



• Market simulation: Quantum models can simulate multiple market scenarios simultaneously, enhancing predictive accuracy.

*Example:* Goldman Sachs and JP Morgan Chase have partnered with quantum companies to explore QML use cases in derivatives pricing and asset management.

#### 4.3 Optimization Problems in Supply Chain and Logistics

Many logistical challenges involve solving large-scale **combinatorial optimization** problems, which are well-suited to quantum computing.

- **Route optimization:** QML can improve solutions to the traveling salesman and vehicle routing problems.
- **Inventory management:** Predictive models can optimize stock levels, distribution schedules, and production planning.
- **Resource allocation:** Quantum optimization methods help in allocating limited resources across dynamic constraints.

Example: Volkswagen has tested QML algorithms for traffic flow optimization in cities.

#### 4.4 Image and Signal Processing

Quantum models are capable of handling high-dimensional data such as images, audio, and signals using fewer computational resources.

- Quantum convolutional neural networks (QCNNs): Adaptations of CNNs in the quantum domain are under development for image classification and edge detection.
- **Pattern recognition:** QML algorithms can be applied to medical imaging, satellite analysis, and remote sensing for faster and potentially more accurate recognition tasks.

#### 4.5 Natural Language Processing (NLP)

NLP tasks such as text classification, sentiment analysis, and machine translation can benefit from QML's ability to process large vector spaces.

- Quantum word embeddings: Quantum circuits can be used to represent semantic relationships between words in compressed forms.
- **Document classification:** Quantum classifiers may be applied to categorize legal documents, news articles, and user feedback with improved complexity scaling.



#### 4.6 Climate Modeling and Environmental Science

Climate models involve massive datasets with chaotic systems. QML can potentially offer faster simulations and improved forecasting.

- Weather prediction: Hybrid quantum models could process climate data to forecast temperature, rainfall, and extreme weather events.
- **Carbon emission tracking:** QML may assist in optimizing systems to reduce emissions based on predictive patterns.

#### 4.7 Cybersecurity and Anomaly Detection

Quantum-enhanced models can analyze large-scale system logs and detect anomalies in real-time, which is crucial for modern cybersecurity systems.

- Intrusion detection: QML can classify network behavior as benign or malicious.
- Quantum cryptography integration: QML could support adaptive security models that react to evolving threats.

# **5.** Limitations and Challenges

Despite its promising potential, Quantum Machine Learning (QML) is still in a nascent stage and faces numerous technical, theoretical, and practical limitations. Many of the claimed advantages of QML remain largely theoretical or confined to idealized scenarios. This section explores the key barriers that currently hinder the widespread adoption and effectiveness of QML.

# 5.1 Hardware Limitations

One of the most significant constraints in QML is the immaturity of current quantum hardware.

- **Qubit Quality and Count:** Most available quantum systems are limited to tens or hundreds of noisy qubits. Practical QML applications may require thousands or more.
- Error Rates and Decoherence: Qubits are highly susceptible to noise and decoherence, which leads to errors during computation. Error correction schemes are still in early development and often require impractically large overhead.
- Limited Circuit Depth: Due to noise and decoherence, the number of operations (gates) that can be performed in sequence is constrained, limiting the expressiveness of quantum models.

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# 5.2 Data Encoding Bottlenecks

Encoding classical data into quantum states—a process known as **quantum feature mapping or state preparation**—is often a computational bottleneck.

- Scalability: Efficiently encoding large datasets into quantum circuits remains a major challenge. In some cases, the encoding cost may outweigh the potential quantum advantage.
- Fidelity and Noise: High-fidelity data encoding requires precise control, which is difficult to achieve on current noisy devices.

# 5.3 Lack of Quantum Advantage in Practice

While some QML algorithms offer **theoretical speedups**, there is limited empirical evidence of **real-world quantum advantage** over classical machine learning.

- **Benchmarking Issues:** Comparing quantum and classical performance fairly is difficult due to differences in architecture, precision, and data representation.
- **Task Suitability:** Many practical ML problems are not known to benefit significantly from quantum algorithms, especially when classical models are highly optimized.

# 5.4 Algorithmic Immaturity and Instability

QML is still evolving as a field, and many algorithms suffer from unresolved issues related to stability, scalability, and convergence.

- **Training Instability:** Variational quantum circuits may face barren plateaus—regions in parameter space where gradients vanish—making optimization difficult.
- **Hyperparameter Tuning:** Similar to classical ML, QML models require careful tuning, but the parameter space is less intuitive and harder to navigate.
- Lack of Standardization: There is a lack of mature, standardized frameworks and benchmarks, which hampers reproducibility and collaboration.

# 5.5 Integration with Classical Systems

In practice, most QML implementations are hybrid, combining quantum and classical components.

- **Communication Overhead:** Data transfer between quantum and classical systems introduces latency and complexity.
- Workflow Complexity: Integrating QML into existing pipelines often requires significant adaptation, especially when dealing with legacy systems or real-time constraints.



# 5.6 Interpretability and Explainability

Quantum models are inherently probabilistic and operate in high-dimensional complex spaces, making them **hard to interpret**.

- **Black-box Nature:** Understanding how a quantum model makes decisions is challenging, which raises concerns in sensitive domains such as healthcare or finance.
- Lack of Tools: Classical ML has a rich set of explainability tools (e.g., SHAP, LIME), but their quantum equivalents are still under development.

# 5.7 Resource and Accessibility Constraints

Developing and testing QML algorithms require access to quantum hardware or simulators, which can be expensive and limited.

- Cloud Access Limitations: Publicly available quantum computing platforms often have queue times, usage caps, and limited qubit counts.
- Skill Gap: QML development requires expertise in both quantum physics and machine learning a rare combination that limits workforce scalability.

# 6. Future Directions and Research Opportunities

As quantum computing matures, Quantum Machine Learning (QML) is poised to become a transformative technology across industries and scientific disciplines. However, realizing its full potential will require significant advancements across multiple fronts. This section outlines promising research directions and emerging opportunities that can shape the future of QML.

# 6.1 Development of Scalable Quantum Hardware

A critical enabler of progress in QML is the development of more powerful and stable quantum hardware.

- Fault-Tolerant Quantum Computers: Building large-scale quantum systems with error correction will allow execution of deeper circuits and more complex algorithms.
- Hardware Specialization: Just as GPUs revolutionized classical ML, quantum hardware optimized for QML tasks (e.g., variational circuits, linear algebra operations) could significantly accelerate performance.
- Quantum RAM (qRAM): Efficient quantum memory architectures are needed for scalable data storage and retrieval within quantum systems.

# 6.2 Advanced Quantum Algorithms for ML

The design of novel algorithms specifically tailored to quantum environments is a fertile area for research.

- **Hybrid Model Innovation:** Further development of efficient hybrid quantum-classical architectures that can dynamically balance workloads between processors.
- Noise-Resilient Algorithms: Creation of algorithms that are robust to the noise and imperfections of near-term devices (NISQ era).



• **Quantum Kernel Methods:** Exploration of more expressive and efficiently computable quantum kernels for classification and regression tasks.

# 6.3 Data Encoding and Representation Techniques

Improving how data is mapped into quantum states is essential for QML scalability.

- Efficient Encoding Schemes: Research into low-overhead, high-fidelity quantum feature maps that retain data structure and meaning.
- Quantum Embeddings: Embedding classical datasets into quantum Hilbert spaces in a way that enhances learning performance.
- **Compression and Sparsity:** Investigating quantum approaches for sparse and compressed data representation to reduce resource demands.

#### 6.4 Interpretability and Explainability in QML

As quantum models are deployed in high-stakes areas, explainability will be key for trust and transparency.

- Quantum Explainability Tools: Development of tools to visualize and interpret the inner workings of quantum circuits and decisions.
- Quantum Fairness and Ethics: Ensuring QML systems uphold ethical standards, avoid bias, and maintain transparency, particularly in domains like healthcare, law, and finance.

#### 6.5 Benchmarking and Standardization

To foster meaningful comparisons and reproducibility, standardized benchmarks and datasets are needed.

- **Benchmark Suites:** Community-agreed performance metrics and datasets for QML tasks across classification, regression, clustering, etc.
- **Performance Evaluation:** Clear criteria for measuring quantum advantage in both synthetic and real-world scenarios.

#### 6.6 Integration into Practical Workflows

Research should focus on embedding QML models into end-to-end ML systems and real-world infrastructures.

- **QML Toolchains:** Building integrated pipelines that support model development, training, validation, deployment, and monitoring.
- **Cross-Domain Applications:** Developing domain-specific QML toolkits (e.g., for bioinformatics, logistics, or smart cities).
- AutoQML: Analogous to AutoML in classical contexts, AutoQML systems could automate the design and tuning of quantum circuits for learning tasks.



#### 6.7 Education and Workforce Development

Building a skilled workforce is critical for QML adoption.

- **Interdisciplinary Training:** Curricula that bridge quantum physics, computer science, and machine learning to cultivate future researchers and developers.
- **Open-Access Learning Platforms:** Expanding access to simulators, notebooks, and community-supported tutorials to lower the entry barrier.

#### 6.8 Policy, Ethics, and Governance

As QML moves toward real-world deployment, thoughtful policy and regulation will be essential.

- **Data Privacy:** Quantum models that access sensitive data must comply with regulations like GDPR, HIPAA, etc.
- **Regulatory Frameworks:** Creating ethical guidelines and governance structures for QML development and deployment.

# 7. Conclusion

Quantum Machine Learning (QML) represents a compelling convergence of quantum computing and artificial intelligence, offering the potential to redefine how we process, analyze, and learn from data. By leveraging the principles of quantum mechanics—such as superposition, entanglement, and quantum parallelism—QML algorithms can, in theory, address problems that are intractable for classical computing systems. Over the past few years, significant strides have been made in developing quantum algorithms for machine learning tasks, exploring applications across diverse domains such as drug discovery, financial modeling, logistics, image analysis, and cybersecurity.

However, despite its promise, QML remains in an early experimental stage, hindered by limitations in current quantum hardware, data encoding challenges, and algorithmic instability. The absence of fault-tolerant quantum machines, the difficulty of maintaining quantum coherence, and the lack of standardized frameworks make it difficult to achieve practical quantum advantage today. Moreover, issues around interpretability, scalability, and integration with classical systems highlight the complexity of transitioning QML from research to real-world applications.

Looking ahead, the field is ripe with research opportunities. Advancements in noise-resilient quantum algorithms, efficient data encoding, hardware improvements, and explainable quantum models will be crucial to unlocking QML's full potential. As quantum devices become more powerful and accessible, and as interdisciplinary efforts grow, QML is expected to evolve from theoretical promise to practical utility—transforming industries and reshaping the future of machine learning.



In conclusion, Quantum Machine Learning is not merely a futuristic concept but a foundational area of research poised to play a significant role in the next era of computing. Continued exploration and responsible innovation in this field will be essential for realizing its transformative impact while navigating its inherent challenges.

# 8. References

- 1. Biamonte, J., Wittek, P., Pancotti, N., Rebentrost, P., Wiebe, N., & Lloyd, S. (2017). *Quantum machine learning*. **Nature**, 549(7671), 195–202. <u>https://doi.org/10.1038/nature23474</u>
- Schuld, M., Sinayskiy, I., & Petruccione, F. (2015). An introduction to quantum machine learning. Contemporary Physics, 56(2), 172–185. https://doi.org/10.1080/00107514.2014.964942
- Cerezo, M., Arrasmith, A., Babbush, R., Benjamin, S. C., Endo, S., Fujii, K., ... & Coles, P. J. (2021). Variational quantum algorithms. Nature Reviews Physics, 3(9), 625–644. https://doi.org/10.1038/s42254-021-00348-9
- 4. Benedetti, M., Lloyd, E., Sack, S., & Fiorentini, M. (2019). *Parameterized quantum circuits as machine learning models*. Quantum Science and Technology, 4(4), 043001. https://doi.org/10.1088/2058-9565/ab4eb5
- Havlíček, V., Córcoles, A. D., Temme, K., Harrow, A. W., Kandala, A., Chow, J. M., & Gambetta, J. M. (2019). Supervised learning with quantum-enhanced feature spaces. Nature, 567(7747), 209–212. https://doi.org/10.1038/s41586-019-0980-2
- Rebentrost, P., Mohseni, M., & Lloyd, S. (2014). Quantum support vector machine for big data classification. Physical Review Letters, 113(13), 130503. <u>https://doi.org/10.1103/PhysRevLett.113.130503</u>
- Liu, Y., Arunachalam, S., & Temme, K. (2021). A rigorous and robust quantum speed-up in supervised machine learning. Nature Physics, 17(9), 1013–1017. https://doi.org/10.1038/s41567-021-01287-z
- 8. Schuld, M., Bocharov, A., Svore, K. M., & Wiebe, N. (2020). *Circuit-centric quantum classifiers*. **Physical Review A**, 101(3), 032308. <u>https://doi.org/10.1103/PhysRevA.101.032308</u>
- 9. Preskill, J. (2018). *Quantum Computing in the NISQ era and beyond*. **Quantum**, 2, 79. https://doi.org/10.22331/q-2018-08-06-79
- 10. Mitarai, K., Negoro, M., Kitagawa, M., & Fujii, K. (2018). *Quantum circuit learning*. **Physical Review A**, 98(3), 032309. https://doi.org/10.1103/PhysRevA.98.032309